

**SUBJECT : MATHEMATICS (SET-I)**

Time : 3 Hrs.

M.M.: 100

**General Instructions :**

- (i) All questions are compulsory.
- (ii) Read all the questions very carefully and answer precisely.
- (iii) In Section-A, each question is of 1 mark.
- (iv) In Section-B, each question is of 4 marks.
- (v) In Section-C, each question is of 6 marks.
- (vi) Use of calculators is not permitted.

**SECTION-A**

- Q1. Find the domain of the function  $f(x) = \frac{1}{x^2 + 2}$
- Q2. Find the derivative of  $f(x) = x \cdot \sin x$
- Q3. Express  $\left(\frac{7\pi}{12}\right)^c$  into degree measure.
- Q4. Write the number of elements in the power set of  $\{-1, 0, 1\}$
- Q5. If  $A = \{D, E, F, G, H\}$  be the set of people working against child labour and  $B = \{E, C, D, G, H\}$  be the set of people working against child marriage. Find  $A \cap B$ .
- Q6. Solve the inequality :  $4x - 3(1 - 2x) \geq 6$
- Q7. Define a relation R on the set N of natural numbers by  $R = \{(x, y) : y = x + 5, x \text{ is a natural number less than } 4; x, y \in N\}$ . List its elements.

Q8. Find principal solutions of  $\cot x + \sqrt{3} = 0$

Q9. If  ${}^nC_5 = {}^nC_7$ , find n.

Q10. Evaluate  $\lim_{x \rightarrow 3} \frac{\frac{1}{x} + \frac{1}{3}}{x + 3}$

### SECTION-B

Q11. (i) If in two circles, arcs of the same length subtend angles  $75^\circ$  and  $120^\circ$  at the centre, find the ratio of their radii.

(ii) Find middle terms in the expansion of  $\left(x - \frac{1}{x^2}\right)^9$

Q12. Using principle of mathematical induction prove that

$$\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right) \dots \left(1 - \frac{1}{n+1}\right) = \frac{1}{n+1} \quad \text{for all}$$

$n \in \mathbb{N}$ .

Q13. Solve :  $2 \cos^2 x + 3 \sin x = 0$

Q14. Out of 20 members in a family, 11 like to donate money to the old age homes and 14 like to donate clothes to the old age homes. Assume that each one likes to donate at least one of the two things. How many like to donate only money and not clothes?

Q15. Find the term independent of  $x$  in the expansion of

$$\left(\sqrt[3]{x} + \frac{1}{2\sqrt[3]{x}}\right)^{18}, \quad x > 0$$

Q16. Prove that  $\tan 4x = \frac{4 \tan x (1 - \tan^2 x)}{1 - 6 \tan^2 x + \tan^4 x}$

Q17. If  $\tan x = \frac{3}{4}$ ,  $\bar{\wedge} < x < 3\bar{\wedge}/2$ , find the value of  $\sin \frac{x}{2}$ ,  $\cos \frac{x}{2}$

and  $\tan \frac{x}{2}$ .

Q18. Evaluate  $\lim_{x \rightarrow 0} \left( \frac{\tan x - \sin x}{x^3} \right)$

Q19. Find the domain and range of the real function  $f$  defined as

$$f(x) = \sqrt{x^2 - 16}$$

Q20. How many numbers greater than 1000000 can be formed by using the digits 1, 2, 0, 2, 4, 2, 4?

Q21. Draw the graph of the function  $f(x) = \cos x$ . Also write its range and domain.

Q22. The English alphabet has 5 vowels and 21 consonants. How many words with two different vowels and 2 different consonants can be formed from the alphabet?

### SECTION-C

Q23. Using principle of mathematical induction prove that  $x^{2n} - y^{2n}$  is divisible by  $x + y$ ,  $\forall n \in \mathbb{N}$ .

Q24. Solve the following inequalities graphically

$$x - 2y \leq 3, \quad 3x + 4y \geq 12, \quad x \geq 0, \quad y \geq 1$$

Q25. Prove that

$$\cos^2 x + \cos^2 \left( x + \frac{\bar{\wedge}}{3} \right) + \cos^2 \left( x - \frac{\bar{\wedge}}{3} \right) = \frac{3}{2}$$

Q26. Find the derivative of  $\sin(x + 1)$  by using first principle.

Q27. The coefficients of 3 consecutive terms in the expansion of  $(1 + a)^n$  are in the ratio 1:7:42. Find n.

Q28. A committee of 5 is to be formed out of 6 men and 4 ladies. In how many ways can this be done, when

- (i) exactly 2 ladies are included;
- (ii) atleast 2 ladies are included; and
- (iii) atmost 2 ladies are included.

Q29. (i) Evaluate  $\lim_{x \rightarrow 0} \left( \frac{x \sin 5x}{\sin^2 3x} \right)$

(ii) If  $\lim_{x \rightarrow 2} \frac{x^n - 2^n}{x - 2} = 80$ , where n is a positive integer,

find all possible values of n.

(iii) Find the derivative of  $\frac{\sin x + \cos x}{\sin x - \cos x}$  w.r.t. x.